Grid Current Modulation

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SYNOPSIS: The term grid current modulator is used to describe those vacuum tube circuits in which modulation is initially produced in the grid circuit of a three-electrode vacuum tube due to the non-linear grid current-grid voltage relation. Comparison with a representative plate current modulator using the same tubes and the same plate potential shows that by modulating at maximum efficiency in the grid circuit and using the plate circuit solely for amplification, the maximum power output is increased about eight times, the power efficiency is increased about five times and the ratio of sideband output to signal input is increased approximately three times. Under these conditions more carrier input power is needed for the grid than for the plate modulator. This improved performance has been made possible by a detailed study of the fundamental processes involved and by a design of the tubes and associated equipment, such as transformers and filters, to permit these fundamental processes to operate to their best advantage.

Normally modulation is also produced in the plate circuit which is shown to be out of phase with that produced in the grid circuit. By inserting high impedances to the input frequencies in the plate circuit, plate circuit modulation is prevented, and the reduction of grid circuit sideband is likewise avoided. By including in the grid circuit an impedance which is high to the desired sideband frequencies, the maximum grid sideband voltage is obtained. In this way the power and modulating efficiencies of the tube circuit are made maximum.

Where modulation occurs only in the plate circuit of a tube, the sideband amplitude is proportional to the product of the amplitudes of the input frequencies when these amplitudes are small. In the present type of grid current modulator the sideband amplitude is proportional to the smaller of the two input amplitudes provided the ratio between these is greater than about 3/2. This feature makes the modulator particularly valuable in communication systems.

The article concludes with an application of the fundamental principles involved to an experimental carrier telephone system in which the operating features of tubes, filters, and transformers are discussed.

INTRODUCTION

BECAUSE of the extensive application of vacuum tube modulators in systems of carrier communication, they constitute an important tool in the hands of telephone engineers. As such they have justified extensive laboratory investigation. The purpose of this paper is to discuss some of the properties of a type of modulator utilizing the non-linear relation existing between grid voltage and grid current and the advantages which recent laboratory investigations indicate that it may possess. Further studies are in progress to determine the conditions under which it can be employed practically.

There are two distinct classes of vacuum tube modulators which may be designated for convenience as grid and plate types, according to the circuit in which the modulation is initially produced, although some modulators may involve both circuits. As an example of the plate

type we might mention the coupled-plate or Heising modulator which has found extensive application in radio transmitters, in which the plate circuit of an oscillating tube is coupled to the plate of another tube through which the signal is introduced, modulation taking place ordinarily in the plate circuit of the oscillating tube. A carrier frequency amplifier is sometimes used in place of the oscillator. Another type of plate modulator, due in principle to van der Bijl, which has found extensive application in carrier telephone systems, applies the signal and carrier to the grid of the modulator, so that the two components are amplified in common before being modulated. As examples of the grid modulator there are the grid leak and condenser type used almost universally for radio reception, together with the type which forms the subject of the present paper, employing a generalized impedance in the grid circuit. The three last-mentioned modulators may incidentally produce modulation in both plate and grid circuits. This ordinarily acts to reduce the overall efficiency as well as to introduce other undesirable features of operation, so that in the design of the grid current modulator we have been led to minimize modulation in the plate circuit, operating the grid circuit as a modulator and the plate circuit purely as an amplifier.

The criteria of usefulness of modulators include some usually placed upon vacuum tube apparatus in general, together with those peculiar to frequency change; some of most importance are modulating gain and level, plate power efficiency, quality, stability, input and output impedances, and carrier suppression. These will be taken as a basis for discussing the operation of the grid current modulator and comparing it with that of the other types mentioned above. The modulating gain usually expressed in transmission units (T. U.) represents the ratio of the power output of a single sideband to the power input of the signal which produces it. It is a function of both carrier and signal amplitudes, usually decreasing at high amplitudes. This decrease in gain should not be too rapid or the modulated output power at sufficiently large signal amplitudes may actually decrease as the signal is increased, and lead to prohibitive distortion. Another aspect of the question relates to the maximum modulated power attainable. The signal amplitude fluctuates within wide limits in course of operation and it becomes desirable to limit its effects, so that the resultant modulated potentials may not disturb the operation of associated equipment. This may be accomplished in modulators in which the sideband output approaches an asymptotic maximum as the signal is increased, better than in those which pass through a maximum in the operating range. A knowledge of the signal amplitude and the

modulating gain corresponding to it is sufficient for a determination of the output amplitude or output level which is of prime importance in matters relating to noise and interference. Other significant factors from the standpoint of noise and interference are the closeness with which line and connecting apparatus impedances are matched since this determines the amount of reflection of an incident wave,[1] and the extent to which carrier current is transmitted to the line (carrier leak) in carrier suppression systems.

It is highly desirable of course to have the efficiency of energy conversion from the plate battery to the sideband output power as high as possible, since the amount of power supplied to the plate circuit is thereby minimized, and the necessary power capacity of the plate supply is reduced. Another kind of plate efficiency in which we are sometimes interested is the efficiency of energy transfer from the plate supply to the external plate impedance. This tells us the amount of energy dissipated by the plate of the tube and fixes its structure; it differs from the first efficiency only when other current components than the sideband flow in the plate circuit. Inasmuch as we shall deal in the following with low power tubes which have ample load capacity, only the first-mentioned efficiency is important.

We are also interested ordinarily in the quality of the system. This is determined in large part by the width of the transmitted frequency band, by the presence of new interfering frequencies introduced by the process of modulation, and by the linearity of the modulated output in terms of the signal input. The first and last conditions are equivalent to the requirements that the modulating gain be maintained both over the ordinary range of signal input amplitudes, and over the frequency band essential for good signal reproduction. Finally the system as a whole is required to have a high degree of stability, so that ordinary variations of battery potentials, or even the replacement of a tube by another of the same type, will not impair the operation of the system.

The specific forms of grid current modulator with which we shall be concerned as an application of the theory are those adapted to carrier current telephony, in which the carrier current is suppressed and a single sideband transmitted. Comparison with a representative plate current modulator using the same tubes at the same plate potential shows that, by modulating at maximum efficiency in the grid circuit and using the plate circuit solely for amplification, the maximum power output is increased eight times, the power efficiency is increased five times, and the ratio of sideband output to signal input is increased

[1] The reflection is measured by the quotient of the difference by the sum of the two connected impedances; this is known as the reflection coefficient.

approximately three times. From these figures it is evident that the space current or plate power supplied the grid modulator is sixty per cent greater than it is for the plate modulator, and that this greater power is utilized five times more efficiently. Under these conditions more carrier input power is needed for the grid than for the plate modulator. Where the carrier oscillator employs a tube of the same type as that used in the modulators, sufficient carrier power is, however, available. This improved performance has been made possible by a detailed study of the fundamental processes involved, and by a design of the tubes and associated equipment, such as transformers and filters, to permit these fundamental processes to operate to best advantage.

In view then of the close interdependence of the circuit elements, we shall start with a discussion of the theory as developed for the simplest circuits, and accompany it by approximate mathematical analyses wherever it appears profitable. No rigorous mathematical treatment appears to be possible or even desirable because of its complexity in the general case, and the sole purpose of our approximate analyses is to help in building up a physical picture of the operation of modulators. With the theoretical conclusions in mind, the characteristics of tubes, transformers, retard coils, balanced circuits, and filter networks which are important in this connection are examined, and the performance of the complete carrier telephone modulator circuit is covered in some detail. The theoretical conclusions are not limited to the carrier telephone modulator which has been used simply for illustrative purposes; as a matter of fact the same principles have been found operative in different types of circuits over a wide frequency range. It should be noted that we have not attempted to combine the oscillating and modulating functions in a single circuit as is sometimes done, but have maintained these circuits distinct from one another, so that the best performance of each may be realized.

Theory of Vacuum Tube Modulation

In a broad sense the same general phenomena are involved in both grid and plate modulation, since modulation is produced when an impedance is varied in accordance with the amplitudes of the modulating potentials, a condition true of both circuits under appropriate conditions. Thus conductive grid current is suppressed at negative potentials in the high vacuum tubes which we employ, and it flows when the grid potential is positive, the grid impedance depending upon the amplitude of the grid potential.

We can obtain a qualitative idea of the situation when we consider

the grid circuit connected to an a.c. generator in series with a high resistance of the order of a megohm, and with the plate circuit connected to a resistance of the same order of magnitude as the internal plate resistance. With a negative grid the input impedance is mainly capacitive and at low frequencies nearly the entire applied e.m.f. exists across the grid. At positive potentials however, when conductive grid current flows and the grid resistance drops to something like 10,000 ohms, by far the greater part of the drop is taken up in the external resistance so that the positive lobe of the grid potential wave is distorted. This distortion is equivalent to modulation since it implies the presence of new frequencies. As a result of the varying reaction of the tube then, modulation voltages are built up across the grid-filament path. These potentials are amplified in the plate circuit where the applied wave suffers further distortion due to the non-linear relation between plate current and grid potential, which in a similar way gives rise to plate modulation. Evidently plate modulation would be alone effective if the external grid resistance were made small.

General Relations

The production of modulated currents or potentials is characteristic of any device in which the relation between instantaneous values of current and voltage is not a linear one. Theoretically, such a relation can be represented to any required degree of accuracy by an equation of the form

$$i = a_0 + a_1 v + a_2 v^2 + a_3 v^3 + \cdots, \tag{1}$$

in which i represents the current through the device and v the potential drop across it. Now suppose a voltage wave which includes two components of frequency $p/2\pi$ and $q/2\pi$ respectively to be impressed on the non-linear element

$$v = P \cos pt + Q \cos qt. \tag{2}$$

If we substitute this expression in eq. 1 for v, the current wave is found to include components of the two original or fundamental frequencies, together with new frequencies produced by the non-linear element:

$$\begin{aligned} i = i_0 &+ i_p \cos pt + i_q \cos qt \\ &+ i_{2p} \cos 2pt + i_{2q} \cos 2qt \\ &+ i_+ \cos (p + q)t + i_- \cos (p - q)t \\ &+ i_{3p} \cos 3pt + i_{3q} \cos 3qt \\ &+ i_{2p+q} \cos (2p + q)t + i_{2p-q} \cos (2p - q)t \\ &+ i_{p+2q} \cos (p + 2q)t + i_{p-2q} \cos (p - 2q)t + \cdots, \end{aligned} \tag{3}$$

in which the coefficients i_k involve the characteristic constants of the tube, together with the applied potential amplitudes:

$$\begin{aligned} i_0 &= a_0 + a_2(P^2 + Q^2)/2 + \cdots, \\ i_p &= a_1P + 3a_3P(P^2 + 2Q^2)/4 + \cdots, \\ i_q &= a_1Q + 3a_3Q(Q^2 + 2P^2)/4 + \cdots, \\ i_{2p} &= a_2P^2/2 + \cdots, \\ i_{2q} &= a_2Q^2/2 + \cdots, \\ i_+ &= i_- = a_2PQ + \cdots, \\ &\vdots \end{aligned} \tag{4}$$

The new frequencies produced are made up of sums and differences of integral multiples of the two original frequencies, and an inspection of eq. 3 shows that the frequency of any component may be put in the form

$$|mp \pm nq|/2\pi \qquad m, n = 0, 1, 2.\cdots$$

It is convenient to designate the sum of the two numbers m and n as the order of a wave component, so that the frequencies $2p/2\pi$, $2q/2\pi$, and $(p \pm q)/2\pi$ are products of the second order. The last of these serves as the basis for the operation of all present [2] carrier systems, and the rôle of any modulator is therefore to produce one or both of these components which are known as side frequencies, or as sidebands when the signal wave is made up of a band of frequencies. Now by repeating the modulating process, but this time with the frequencies $(p + q)/2\pi$ or $(p - q)/2\pi$, or both, together with the component of frequency $p/2\pi$, designated as the carrier wave, it is well known that one of the resultant second order products has the frequency of the original signal $q/2\pi$. This second or receiving modulator, sometimes designated as a demodulator or detector, is separated from the first one by a transmitting medium and frequency-selective apparatus so that only the desired components may be transmitted and received. The impedance-frequency characteristics of these elements with which the modulators are associated are of prime importance in determining the modulation, as is best brought out by a discussion of some approximate mathematical analyses which follow.

Modulator Circuit, Small Alternating Potentials

We shall consider the current-voltage characteristic of a vacuum tube to be given, to sufficient accuracy for our purposes, by the first

[2] Higher order products, such as $(2p \pm q)/2\pi$ have been equally well employed but we shall confine our attention here to the usual second order system.

three terms of eq. 1, and shall suppose two generated potentials, of frequency $p/2\pi$ and $q/2\pi$ respectively, to be applied to a tube circuit which includes a series impedance. This impedance Z_k may be a function of frequency as indicated by the subscript k, which refers to the particular frequency at which the impedance is effective. The variable part of eq. 1—the change in current produced by application of the alternating potentials—is clearly

$$J = a_1 v + a_2 v^2. \tag{5}$$

The potential drop across the tube is that impressed minus the Zi drop or

$$v = \Sigma(E_k - Z_k J_k) \tag{6}$$

the summation extending over all current components. As a first approximation to the fundamental currents we may neglect the non-linear term $(a_2 v^2)$ in eq. 5 to obtain

$$\begin{aligned} J_p &= a_1 E_p/(1 + a_1 Z_p), \\ J_q &= a_1 E_q/(1 + a_1 Z_q). \end{aligned} \tag{7}$$

Using these solutions we can obtain a second approximation [3] taking into account the non-linear term which we neglected for the first approximation. Thus

$$\begin{aligned} J_p + J_q + J_2 = a_1 \left(\frac{E_p}{1 + a_1 Z_p} + \frac{E_q}{1 + a_1 Z_q} - Z_2 J_2 \right) \\ + a_2 \left(\frac{E_p}{1 + a_1 Z_p} + \frac{E_q}{1 + a_1 Z_q} \right)^2, \end{aligned} \tag{8}$$

in which the subscript 2 indicates the second approximation. By squaring the second member of eq. 8 it is observed that the second approximation includes direct current, second harmonics of the two impressed frequencies $p/2\pi$ and $q/2\pi$, and the second order sidebands. For these last we obtain the expression

$$J_{\pm} = \frac{a_2}{(1 + a_1 Z_{\pm})} \frac{E_p E_q}{(1 + a_1 Z_p)(1 + a_1 Z_q)}. \tag{9}$$

The sideband potential across the variable element is clearly $Z_+ J_+$ or $Z_- J_-$ according to the particular sideband in which we are interested. Thus

$$V_- = -\frac{a_2}{a_1^2} J_p J_q \frac{Z_-}{1 + a_1 Z_-} = -\frac{a_2}{a_1^2} J_p J_q \frac{Z_-}{R_0 + Z_-}, \tag{10}$$

where $1/a_1$ is equivalent to R_0, the plate resistance of the tube.

[3] Carson, "A Theoretical Study of the Three Element Vacuum Tube," *Proc. I. R. E.*, 1919.

These equations are equally applicable to grid and to plate circuits provided the potentials are small and the operating region is expressible by eq. 5. This is not always the case in practice, and modifications in the above comparatively simple analysis are required, which will be treated below. A number of characteristic features of operation are exhibited by the above analysis however and these we proceed to discuss.

If the preceding treatment is applied to the grid circuit we see that in order to make the sideband potential across the grid a maximum with fixed fundamental currents, the external grid impedance at the sideband frequency must be made large compared to the effective internal resistance of the tube. Further, if the generator potentials and impedances are fixed it follows that the generator resistance should be made to match the internal resistance of the tube at the fundamental frequency—with a transformer, if necessary—in order to make the fundamental currents as large as possible. This conclusion regarding the ratio of grid impedances follows immediately without mathematical analysis if we suppose the source of the higher order products to lie in the variable impedance element so that it may be considered equivalent to the presence of generators of the higher order frequencies. The generator voltage is evidently maximum on open circuit, which agrees with the above statement.

In considering the form of external impedance to use for best results, an inspection of eq. 10 shows that in the quantity $Z_-/(R_0 + Z_-)$, which expresses the ratio of effective grid voltage to generated grid voltage, the sideband impedance Z_- should have a large reactive component. This is illustrated by Fig. 1 which gives the ratio for various relative external impedances having phase angles of 0 and 90° and shows that, with the external impedance fixed in magnitude, the ratio has its greatest value for a pure reactance.

Relative Phase of Grid and Plate Sidebands

If the tube acted as a perfect amplifier of the potentials impressed on the grid, there would be no further distortion and the sideband current in the plate circuit would be obtained by multiplying eq. 10 by $\mu/(Z + R_0)$, where Z and R_0 are the external and internal plate circuit resistances respectively, and μ is the amplification factor which is assumed constant here.[4] Unfortunately this ideal situation does not exist of itself, and modulation of the amplified fundamentals takes place in the plate circuit, producing an additional sideband component to

[4] The distortion due to variable μ as treated by Peterson and Evans in the *Bell System Technical Journal* for July, 1927, represents but a small part of the total in efficient modulators, although it is of importance in high quality amplifiers.

combine with that generated in the grid circuit and amplified in the plate circuit.

Inasmuch as the amplification factor decreases, and the plate impedance increases as the grid potential goes negative, the grid

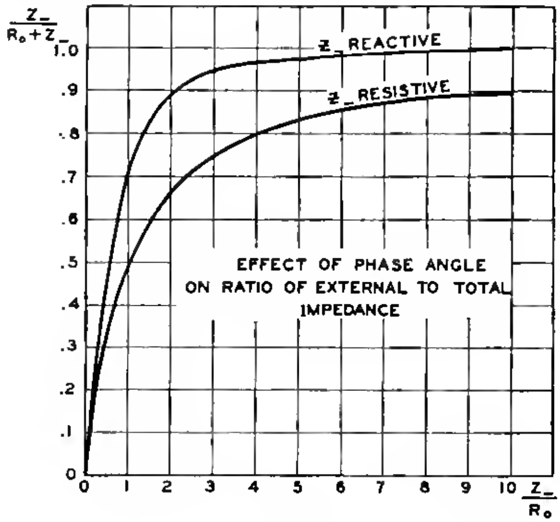


Fig. 1

potential wave is amplified more efficiently on the positive than on the negative lobe, with the result that the plate current wave is limited on one side by the grid current cut-off, and on the other side by plate current cut-off. The second cut-off tends to make the output wave more nearly symmetrical about a horizontal axis; it is therefore equivalent to an increase in the odd order modulation which we do not employ here, and to a reduction of the even order products, one of which—the second order sideband—is used to transmit signal characteristics. It follows that for efficient modulation we must do one of two things—phase the grid and plate products to add, or remove one of the conflicting sources of modulation.

To account for the effect of plate distortion we may apply the same general procedure to the plate circuit as we did to the grid circuit. The plate current-grid potential relation is given as

$$J = b_1 \mu v + b_2 \mu^2 v^2 \tag{11}$$

and the solution for the current components may be written down directly since the problem presents itself in the same form as the grid circuit situation previously considered. Hence if we change the *a*

coefficients to b's, and the Z_k of the grid circuit to the Z_k of the plate circuit, we obtain the expressions

$$\left.\begin{aligned} J_1 &= b_1\mu v/(1 + b_1 Z_1), \\ J_2 &= \frac{b_2}{(1 + b_1 Z_2)}\left(\frac{\mu v}{1 + b_1 Z_1}\right)^2. \end{aligned}\right\} \tag{12}$$

Now it is apparent that each of these two terms contributes something to the sideband frequency—the first by amplification of the grid sideband potential, and the second by modulation of the two fundamental components in the plate circuit. The net sideband current in the plate circuit may accordingly be expressed as

$$J_{\pm} = \left[\left(\frac{\mu b_2}{(1 + b_1 Z_p)(1 + b_1 Z_q)} - \frac{b_1 a_2 Z_{\pm}}{1 + a_1 Z_{\pm}}\right)\right] \left[\frac{\mu E_p E_q}{(1 + b_1 Z_{\pm})(1 + a_1 Z_p)(1 + a_1 Z_q)}\right]. \tag{13}$$

Under normal conditions both grid current and plate current characteristic curves are concave upward, so that b_1, b_2, and a_2 are all positive. The two terms of eq. 13 are then in phase opposition, a condition which is responsible for failure to work certain modulators to fullest advantage. For efficient plate modulators the grid modulation term should be suppressed, which may be accomplished by making the external grid impedance to the modulated product of interest equal to zero, or by keeping the grid potential negative at all times. For efficient grid modulators the plate modulation term should be reduced to a minimum by suppressing the fundamental currents in the plate circuit, in which case Z_p and Z_q are made large. Of course the possibility exists of phasing the two sideband components to add rather than to subtract (arithmetically)—and this, it will be readily seen, is obtained by having the phase angle of the entire plate circuit approach 90° at each fundamental frequency when the grid circuit sideband impedance is large. This condition cannot be met without lowering the amount of plate current modulation, so that the first mentioned plate circuit condition is the more practical one.

Other possibilities of more favorable phasing exist by working within appropriate regions of tube operating characteristics where either b_2 or a_2 becomes negative. Generally speaking, operating points of this nature are not stable with variations in tube potentials, nor are they adaptable to large power outputs approaching the maximum load capacity of the tube. Finally, for straight amplification purposes the two terms of eq. 13 should be made equal and opposite in sign.

In order to test directly the conclusions regarding relative phase of grid and plate modulation products, a circuit was set up which permitted two frequencies to be supplied to the grid of a vacuum tube, the resultant currents of sideband frequency being measured in grid and plate circuits by means of a current analyzer.[5] As shown in Fig. 2 the

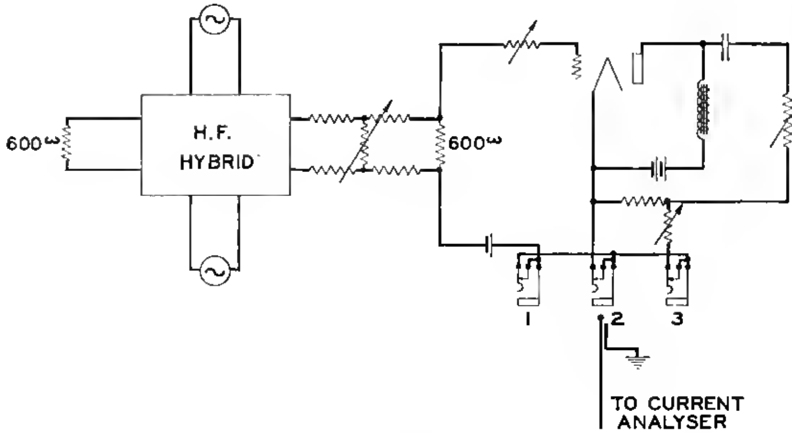


Fig. 2. Test Circuit

grid circuit contains a high external resistance (for producing grid current modulation when conductive grid current flows as previously explained) in series with a "C" battery to vary the relative amounts of grid and of plate modulation. The relative phase of sideband currents produced in grid and plate circuits was calculated from the currents measured separately in jacks No. 1 and No. 3 and their vector sum in jack No. 2. These measurements verified the conclusions drawn from eq. 13,—that with resistances in grid and plate circuits second order modulation products produced in the grid circuit are exactly out of phase with the same frequencies produced in the plate circuit.

The effect of the grid circuit resistance when conductive current flows is of course to limit the positive potentials applied to the grid and so, in effect, to cause the input-voltage—output-current relation of the circuit to be deflected at the upper end more nearly to parallelism with the x-axis than it is for the tube alone. We may therefore consider grid modulation as equivalent to the introduction of a reversed curvature in the operating characteristic. To substantiate this point a tungsten filament tube was used in which the curvature of the lower branch is nearly the same as that of the upper branch, as shown in Fig.

[5] "Analyzer for Complex Electric Waves," by A. G. Landeen, *Bell System Technical Journal*, April, 1927.

3. The plate circuit sideband was measured as a function of the "C" battery with zero grid resistance so that the plate circuit was responsible for the total sideband production; the data are plotted on the

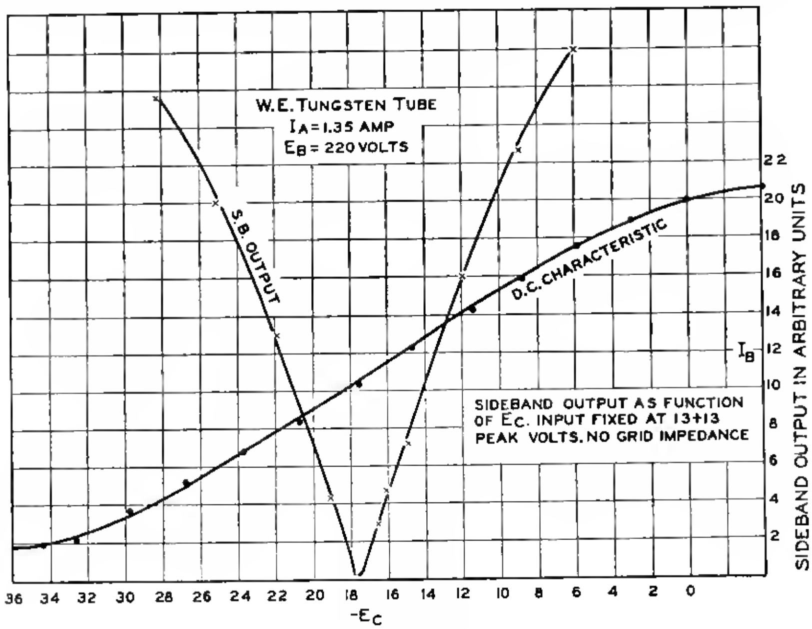


Fig. 3

same Figure. It is seen that the sideband drops nearly to zero when the "C" battery is adjusted so that the input voltage swings symmetrically over the upper and lower branches of the curve. These results could very well be attributed to out-of-phase modulation resulting from reversed curvature. As a matter of fact the algebraic expression for the characteristic involves in general a series of both odd and even powers of applied voltage, but if the axis is taken at the point of symmetry of the characteristic the even powers drop out. Now since the even orders of modulation can be attributed only to the even powers of the static equation, it might be expected that these components would drop to zero.

We may conclude from this discussion that best results will be had in the practical design of grid current modulators, when the external grid impedance at the sideband frequency is made as high as possible and when the impedance to the fundamentals is matched. As to the plate impedances, the situation is the reverse of that existing in the grid circuit since we must have the impedances to the two modulating

frequencies as high as possible, and match the tube impedance at sideband frequencies in order to develop maximum sideband power in the load resistance. It will be observed that with these conditions satisfied, the plate circuit of the tube acts substantially as an amplifier of the sideband produced in the grid circuit, since none is developed in the plate circuit.

Reaction of Sideband Flow on Tube Impedance

In any non-linear system such as the grid circuit or the plate circuit of a tube, the modulation products resulting from the lack of linearity have amplitudes which depend upon one or more of the impressed fundamentals, and react upon the fundamental amplitudes. It follows that the amplitude of any modulation product depends in general upon the amplitudes of all other modulation products, and that the impedance offered to the flow of fundamental depends upon the reaction of the modulation products. Stated otherwise, the amplitude of any one current component depends upon all other components.

This may be demonstrated quantitatively by higher approximations than the two which we have already obtained, in which expressions for the currents are found to contain terms proportional to the sideband voltage across the tube; in fact, if we went to the labor of including a number of distorting components, terms in the fundamental current equation due to their reaction would result. The effects found with a single sideband are simply typical. If, for example, we put (7) and (9) in (8), we get

$$J_p = a_1(E_p - Z_pJ_p) - a_2(E_q - Z_qJ_q)\frac{a_2E_pE_qZ_+}{(1+a_1Z_p)(1+a_1Z_q)(1+a_1Z_+)},$$

and a similar expression for J_q, as second approximations to the fundamentals. These furnish us with a pair of simultaneous cubics in J_p and J_q. When we assume the reaction of the sideband flow on J_q to be small so that eq. 7 remains valid, the above equation becomes linear in J_p,

$$J_p = E_p \Big/ \left(\frac{1}{a_1} + Z_p + \frac{a_2^2}{a_1^4}J_q^2\frac{Z_+}{1+a_1Z_+}\right). \tag{13a}$$

This shows that the impedance in the fundamental path has been increased due to non-linearity by the amount of the last term which may be denoted by ΔZ_p where

$$\Delta Z_p = \frac{a_2^2}{a_1^4}\frac{Z_+}{1+a_1Z_+}J_q^2.$$

A similar expression exists for ΔZ_q when J_p is substituted for J_q. The reciprocal of a_1 will be recognized as the internal resistance of the variable element for small potential variations.

From these

$$J_p^2 \Delta Z_p = J_q^2 \Delta Z_q,$$

so that the two fundamental circuits share equally in the power dissipation due to sideband flow. This means that when the two modulating currents are not of the same amplitude, the smaller current will have the larger resistance change due to sideband flow, and therefore will suffer a greater percentage amplitude change. This discussion serves to emphasize the point that the tube impedances depend upon the impedance-frequency characteristics of the circuit to which the tube is connected, so that this point must be kept in mind in the design and measurement of modulating circuits.

Grid Current Modulator, Large Alternating Grid Potentials

The comparatively simple analysis we have just employed is not capable of very wide application because of the assumed form of the grid current equation. In the practical forms of grid current modulators, from which comparatively large amounts of modulated power are required, the grid potentials are increased and the grid is maintained negative during an appreciable part of the cycle. The above method then becomes too involved to be extended to this case, since a large number of terms would be required for an accurate representation of the tube characteristic. When we have a large external grid resistance, however, as appeared to be desirable from eq. 10, a fairly exact solution for the modulation products can be obtained by another method which is capable of direct application.

If we determine the relation between impressed potential and output current in this particular case we find that on passing from negative to positive potentials, the plate current curve breaks sharply at about zero grid potential, and becomes nearly parallel to the x-axis, as shown in Fig. 4. We can therefore consider the positive lobe of the input wave to be cut off at zero grid potential under these conditions and the problem can be handled analytically.[6]

We are indebted to Mr. F. Mohr for computations on the sideband amplitude as given by eq. 20, of the Appendix, in which the sideband is expressed in terms of a multiple of P, as function of the ratio Q/P. The relationship between these quantities is given as a single-valued function. For our own purposes, however, we have plotted the

[6] Appendix 1.

sideband potential as a function of one of the modulating potentials with the other as parameter, as shown in the dotted lines of Fig. 5. The experimental data are plotted as the full lined curves and appear

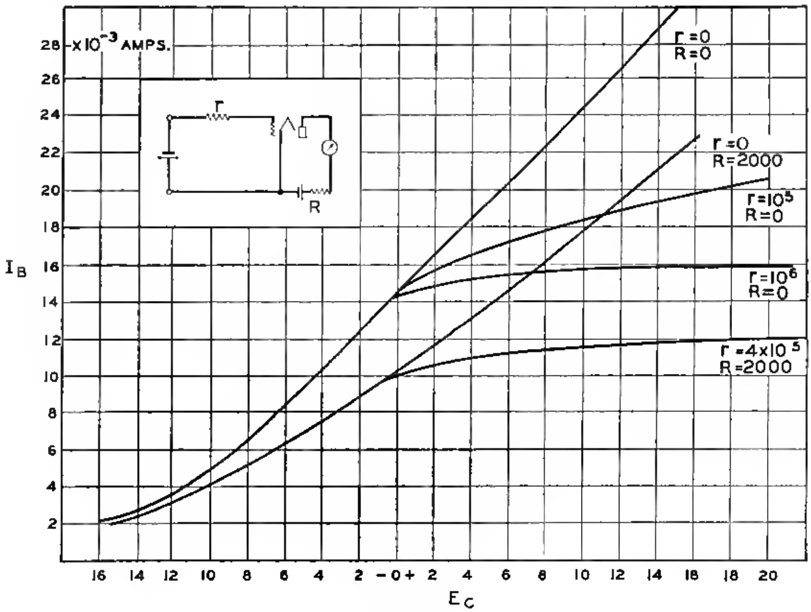


Fig. 4

to be in good agreement with the theory, the divergence being due presumably to the incomplete suppression of the positive lobe in the experimental set-up. The measurements were carried out with the current analyzer as described in connection with Fig. 3, grid current being measured as a function of the two input amplitudes. The sideband grid potential was then determined by multiplying the sideband current by the external grid resistance.

Possibly the most striking thing shown by Fig. 5 is that the sideband amplitude is independent of the larger of the two inputs, when the ratio of one input to the other is made sufficiently great. Hence we must provide sufficient carrier amplitude to insure that the resultant sideband shall be linearly proportional to the impressed signal up to its greatest value so that good quality of speech transmission may be assured.

Our earlier analysis using eq. 5 to represent the grid current characteristic led to a sideband amplitude proportional to the product of the two inputs whereas in this case in which the positive lobe is completely

suppressed, it is proportional to the smaller of the two when the ratio of the two inputs is greater than about 3/2. This proceeds from the fact that eq. 5 must be supplemented by many more terms involving higher powers of the impressed potentials, in order to represent the

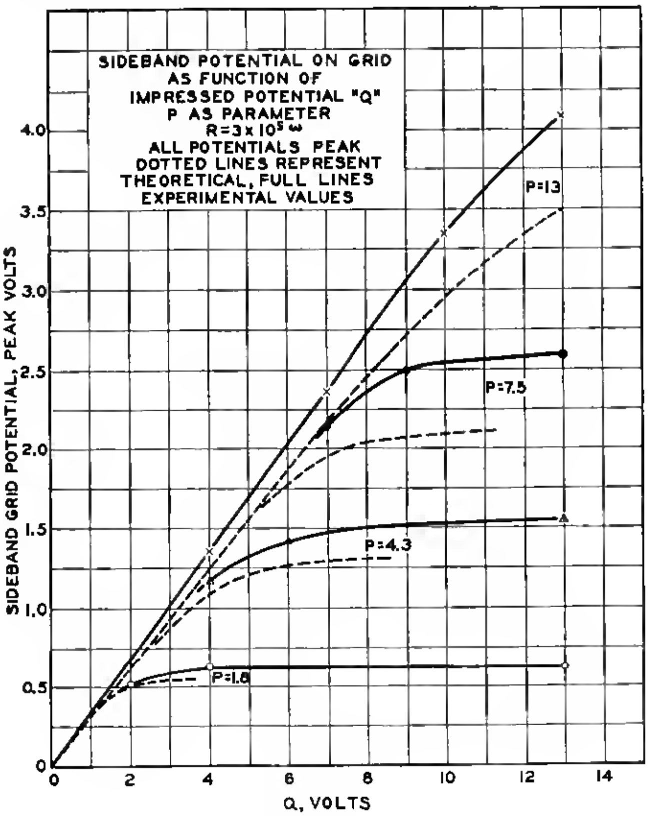


Fig. 5

grid current characteristic to any degree of precision when the grid is driven negative.

The form of the input-output curve is especially valuable for telephony. The relative independence of the larger of the two inputs means that the sideband output will be stable with regard to carrier current variations under the limitations noted. The output approaches a maximum asymptotically, so that the articulation at heavy loads may be expected to hold up better than in those modulating systems in which the output passes through a pronounced maximum. In a system transmitting the carrier, as in radio, and in which a square law

detector is used, the voice output is proportional to the product of the received carrier and sideband. Any change in attenuation, expressed in transmission units (T. U.), between the transmitting and receiving station affects the output current by twice that number of T. U. In the above grid current demodulator, however, the output changes to the extent of the attenuation change, and varies no more than in a carrier suppression system with the carrier locally supplied.

Having determined the grid voltage components, we may now apply the plate circuit coefficients to the grid potential in order to determine the plate current components, just as we did in the previous case. There, it will be recalled, we used a simple representation for the plate current in terms of the grid potential from which amplification and modulation terms were deduced. The same general considerations regarding phase opposition are carried over unchanged.

Limitation of Sideband Output

The above method of treatment is quite satisfactory when the space current is never reduced to zero, but when the grid voltage goes sufficiently negative, precisely the same limitations apply to the plate characteristic equation as applied to the grid equation under similar circumstances, and there exists an additional source of distortion in the plate circuit. In this circumstance the method of expansion in Bessel coefficients cannot readily be used because of the large number of components in the wave subjected to additional distortion, which would lead to prohibitive complication. We may nevertheless obtain a qualitative idea of the result in special cases of interest to us in this connection.

We shall assume that, as we found previously to be advisable, one of the two fundamental currents is substantially suppressed in the plate circuit, so that despite the non-linearity of the plate circuit sideband components are produced only in the grid circuit. We are therefore concerned with the variation of amplification with operating parameters. Now it is clear to start with, that at sufficiently small grid potentials, the entire variation falls within the region of variation of the plate dynamic characteristic so that the result may be written down as in the previous analysis. As the amplitude is increased, the negative end of the grid swing finally has no effect in varying the space current, and the distortion which results tends to limit the magnitude of the amplified components. Hence as the sideband potential on the grid is increased by increasing the applied signal potential and keeping the carrier potential large enough (say one and one half times the signal) so as to get the full efficiency of grid current modulation, the

sideband current in the plate circuit increases linearly with the signal up to a certain point. At this point, which corresponds to the plate current cut-off, the output departs from the linear relation and increases less rapidly. Further increase of the carrier produces no increase in output but a reduction of the output may result because of a greater swing beyond the cut-off point.

Inasmuch as the modulating potentials together with undesired modulated products form a wave having a net amplitude considerably greater than that of the useful sideband, it is clear that the maximum output amplitude can be increased by suppressing the undesired current components thus avoiding the loading and heating effects produced in large part by these other components. A method of attaining this desired result will be treated in connection with balanced circuits. The loading effect may be partially ameliorated very simply since one of the products of modulation is a d.c. component. The presence of series grid resistance means that we have in effect a negative bias applied to the grid which becomes increasingly negative as the input amplitude increases,—just the sort of thing, in other words, to limit sideband production. If, therefore, we use grid reactances instead of grid resistances we can achieve the same degree of modulating efficiency in the two cases at low inputs, and in addition remove effective grid bias, the maximum output power available being increased to a very considerable extent. Of course the insertion of grid reactance changes the details of the conclusions for the grid resistance, but the main features of performance are retained.

When grid resistance is used to provide a high impedance to the sideband, the operation of the grid leak and condenser detector is approached, in respect to the undesirable increase of bias with increase of input. As a consequence the output power is limited at large inputs, although the gain is fairly high at small input amplitudes.

Another point affecting the operation of the grid leak and condenser detector is the plate circuit impedance. According to the conclusions of the above theory for grid current modulation, the output power is increased at large input amplitudes by providing an impedance in the plate circuit which is high to both input frequencies and matches the tube impedance at all desired output frequencies. This conclusion has been verified experimentally at carrier frequencies when operating the tube for maximum output, but is contrary to the usual practice in radio circuits, where the plate circuit impedance to the modulating frequencies is ordinarily made low rather than high compared to the tube impedance. The problem is complicated at radio frequencies by regenerative effects not present to the same degree at the compara-

tively low frequencies used in carrier telephony, and by the comparatively low alternating and battery potentials which raise the relationship of plate and grid voltages to grid current, to importance.

We have now to examine the electrical properties of available circuit elements in the light of our previous analysis, so that their assembly will yield the most favorable results.

Vacuum Tubes

The effect of the shape of the grid-current—grid-voltage curve on the modulating properties of the grid circuit is not as pronounced at large amplitudes as might be expected from experience with plate current modulators at comparatively low amplitudes. As is well known this characteristic of ordinary tubes is much more variable between tubes of the same type than the plate-current—plate-voltage curve. But it has been found that a change of tubes having static grid characteristics varying within wide limits does not vary the modulating gain of a grid current modulator more than one T.U. The reason for this may be seen most easily in the case of an external grid impedance consisting of a pure resistance. If the tube grid resistance were comparatively small for all positive voltages the positive half of the wave would be completely suppressed, and the analysis of Appendix 1 would accurately represent the wave. Even when the tube grid resistance varies considerably it does not alter the wave shape appreciably so long as it remains small compared to the external resistance. This condition may be satisfied with particular ease for large input voltages, and may also be satisfied in a qualitative sense, when reactances are used in place of resistance. The principal effect of a change in grid resistance is then to change the input impedance, which affects the net gain only through the mismatch of impedance at input frequencies.

As a consequence of the tube circuits and range of operating potentials used in the grid current modulator, the details of the grid current characteristics become of relatively small importance and attention is focussed on the functioning of the plate circuit. The plate circuit is used purely for amplification purposes as mentioned above, so that the criteria of usefulness of a tube as a grid current modulator come down ordinarily under the stated operating conditions to the criteria of usefulness of a tube as an amplifier.

Filter and Transformer Networks

Input Filters and Modulating Gain

Since the gain obtainable in a grid current modulator depends primarily on the ratio of external to total grid circuit impedance, it is

necessary to consider how the required high impedance may be obtained in practice. The input transformer must have an impedance looking into the grid side which is high to all sideband frequencies, and must at the same time transmit efficiently all signal input frequencies. A high impedance over the sideband range is best obtained by a filter [7] on the low side of the input coil, care being taken to allow for the effect of the transformer on the filter impedance. In order to determine the actual external grid impedance and to investigate the modification of filter impedance by the input transformer, a high impedance bridge was built in which precautions were taken to prevent errors due to the high impedances involved (up to several megohms). In each case only the end section of the filter adjacent to the modulator was used, since this provided nearly the same impedance as would be given by a complete filter. Low pass filters are used on the input to the modulator and on the output of the demodulator, while band pass filters are used on the output of the modulator and the input of the demodulator. These are to be considered in turn.

Low Pass Filters

The simplest type of low pass filter is the infinity type of section, the impedance characteristics of which are shown in Fig. 6*a*. The filter alone, as shown by the solid lines, has negligible reactance in the transmission band (0–3 K. C.) and practically pure inductance in the attenuated region. The input transformer resonates in the attenuated region when terminated by this filter, as shown by the dotted lines, because of the leakage inductance and distributed capacity of the windings. The resonance peak is quite broad due to the comparatively high a.c. resistance and so covers a considerable frequency range as is shown by the ratio of $Z_-/(R_0 + Z_-)$ in Fig. 6*c*. It may be made to appear at higher frequencies by using a filter with a higher cutoff frequency, and at a lower frequency by replacing the series inductance by a parallel tuned circuit (an m-type section).[8] A new type of filter section developed for certain phases of this work and known as the built-out type [9] has a particularly good impedance characteristic in the attenuated region, as shown by the solid lines of Fig. 6*b*. But as shown by the dotted lines the transformer impedance, when terminated in this type of section, is very much modified by the coil constants. The resulting efficiency as show by the ratio $Z_-/(R_0 + Z_-)$ in Fig. 6*c* is not as good as that of the infinity type section.

[7] For a general discussion of filter impedances and attenuations see Campbell, *Bell System Technical Journal*, November, 1922; Zobel, January, 1923; Johnson and Shea, January, 1925.
[8] O. J. Zobel, *Bell System Technical Journal*, October, 1924.
[9] Devised by T. E. Shea of Bell Telephone Laboratories.

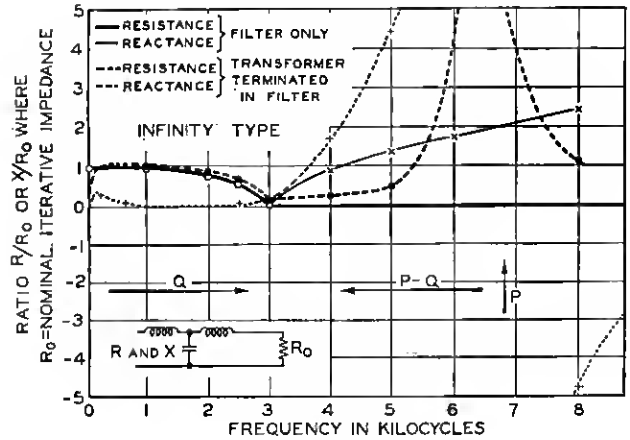


Fig. 6a

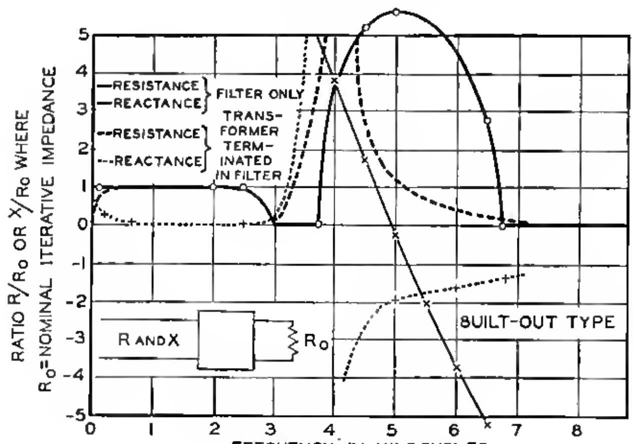


Fig. 6b

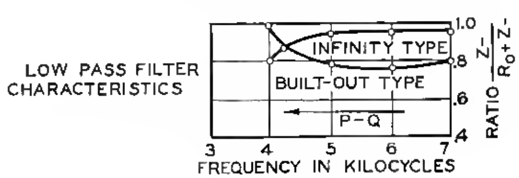


Fig. 6c

Band Pass Filters

The two most important types of band pass filter sections—the confluent and the built-out types—are shown in Fig. 7. The resonance of the confluent section in the voice frequency range does not affect the

ratio of external to total impedance appreciably but the chief defect is in the comparatively low value of this ratio at the higher voice frequencies (2.5 to 2.8 K. C.), although this type of section is satisfactory over the entire voice frequency range at higher carrier frequencies.

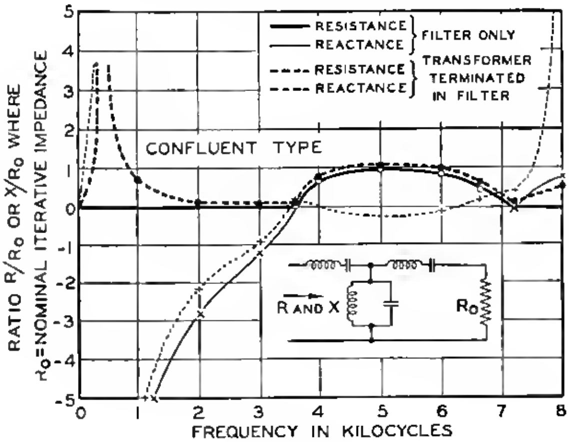


Fig. 7a

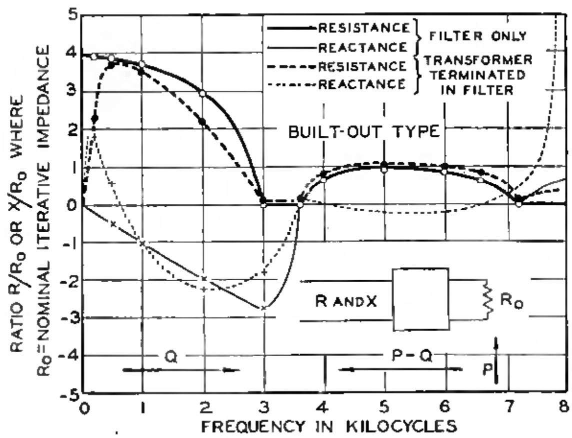
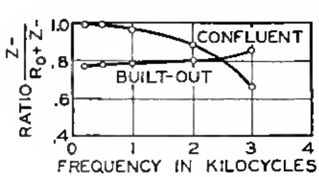


Fig. 7b



BAND PASS FILTER CHARACTERISTICS

Fig. 7c

The built-out band pass filter shown in Fig. 7*b* has a very satisfactory impedance over the voice range and the modifications introduced by the transformer do not seriously affect its efficiency. From the curves of Fig. 7*c* it is evident that the built-out type of section must be used for channels near the voice frequency range but that the confluent type shown in Fig. 7*a* may be used for the higher frequency channels.

The close relation between input filter impedance and modulating gain is illustrated in Fig. 8. Two band pass filters were built having

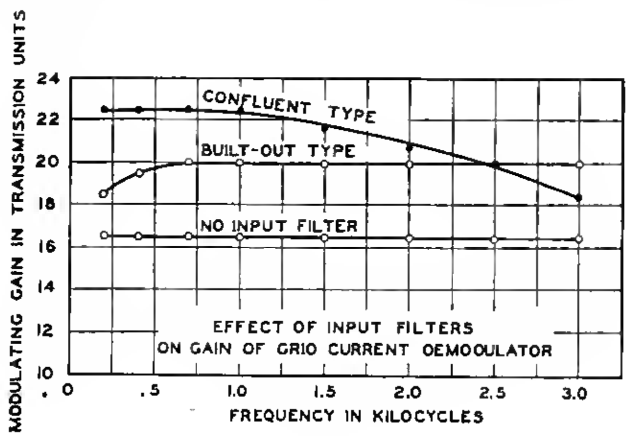


Fig. 8

impedance characteristics approximating the curves shown in Figs. 7*a* and 7*b*. It would then be expected that the modulating gain would be proportional to the ratio $Z_-/(R_0 + Z_-)$. The curves in Fig. 8 show that this is very nearly the case. With no input filter the ratio $Z_-/(R_0 + Z_-)$ would be 0.5 or 6 T.U. less than the maximum possible gain, as is found to be actually the case. This shows that the modulating gain may be calculated for any value of input impedance if it is known for any other value.

Input Impedance

The impedance looking into the low side of the input transformer when the high side is terminated in the grid circuit of a modulator under operating conditions (carrier at normal value) depends on a number of factors, among which the principal variables are the signal and carrier input currents, the input transformer, and the input generator impedance. As might be expected, the input impedance decreases as either carrier or signal amplitude is increased, and the change of impedance with signal amplitude is small when the signal is small compared to the carrier, as is normally the case.

The influence of the input transformer upon input impedance depends not only upon first order, but also upon higher order effects. The first order effect is simply due to the transformer terminated in a network having a linear current-voltage characteristic, which may be calculated from the usual transformer theory. The higher order effect is produced by the effect of the contributions to fundamental frequencies caused by the flow of modulation currents, as discussed in connection with Equations 13*a* and 4. For this reason the impedance of the external grid circuit at other than input frequencies may have a considerable effect on the input impedance. It has been found possible to reduce the reflection from a resistance line to a small value with suitable transformers.

Output Filters and Transformers

The general effect of an output filter or retard coil in the plate circuit with high impedance to all frequencies except the sideband is to increase the output level for large inputs, since the opposing effect of plate modulation is eliminated and the total load capacity of the tube is employed solely in the amplification of the sideband. The output transformer on account of its low ratio has very little effect in altering the impedance-frequency characteristic of the output filter so that we need not enter so thoroughly into the details as we did in the case of input filters.

Output Impedance

The output impedance of a grid current modulator (looking from the line into the output coil) is affected mostly by the transformer ratio and the impedance to carrier in the plate circuit. If the impedance to the carrier frequency is very high, as is usually the case, there will be very little modulation with the carrier in the plate circuit, and neither the carrier input current nor the external output impedance at signal frequencies affects the output impedance appreciably. The reflection may be made quite small over the frequency range without any great difficulty.

Gain-Frequency Characteristic

The problem of obtaining a flat frequency-gain characteristic over the voice range depends upon the attenuation of input and output transformers, the attenuation of filters, and the impedance characteristic of input and output coils when terminated by their respective filters. The transformer attenuation is comparatively small and affects the frequency characteristic mostly at frequencies below 200 cycles. The closer the carrier channels are spaced to each other or to the voice band, the more difficult it becomes to obtain filters with suf-

ficiently sharp cutoff. In most cases a maximum variation of 2 T.U. in the attenuation over the transmitted band is a reasonable figure for a band pass filter. Each transformer and filter tends to increase the attenuation at the edges of the transmitted band more than in the center so that frequencies from 800 to 2,000 cycles are always transmitted with minimum attenuation, which is independent of the frequency-output current characteristic of the modulating elements.

The above consideration of filter attenuation is substantially independent of filter impedance since the latter is determined mostly by the end section. From the previous consideration it is evident that either may have a very pronounced effect, so that in measuring the frequency characteristic of a modulator or demodulator both attenuation and impedance effects must be taken into account. The effects of input and output impedances can be partially separated when the carrier is suppressed because of the fact that the output impedance has but little effect at small inputs and the input impedance has but little effect at large input currents.

Balanced Tube Circuits

The present practice in carrier telephone systems is to suppress the carrier current and one sideband in order to conserve frequency space and to reduce the energy levels and the cross-talk in associated equipment. The elimination of undesired components of a wave may be carried out by two distinct processes,—frequency discrimination by filter networks, and phase discrimination or balance by bridge circuits.[10] Each method is useful and both find places in carrier systems. When the frequency separation between desired and undesired components becomes relatively small, frequency discrimination becomes impractical and expensive. The balance method is used to separate frequencies according to their respective phase relations in two or more similar modulating circuits, the phases of the output components depending on the relative phases of the input currents. Consequently only certain combinations of modulation products can be separated by balance and these only to an extent determined by the balance attainable in transformers and vacuum tubes, both of which are subject to manufacturing variations. Due to the proximity of the carrier and second order sideband frequencies the suppression of carrier current by filter circuits alone is impractical. Balanced circuits must be used for this purpose and in spite of unavoidable variations in tubes and circuits it is usually possible to reduce the carrier on the line to less than five per cent

[10] For an illustration of balanced circuits, reference may be made to U. S. Patent 1,343,306, issued to J. R. Carson.

of its normal unbalanced value. To separate one sideband from the other after the carrier has been suppressed, and to suppress unbalanced components other than the carrier, filter attenuation is customarily employed.

In the usual type of balanced circuit there are two possible input paths with corresponding output circuits, one connected to the two grids in series; and known as the series path; the other to the two grids in parallel, known as the shunt path or midbranch. When carrier is impressed on the midbranch and signal on the series arm—the present arrangement in commercial carrier systems using plate current modulators—we designate the circuit, as a matter of convenience, as the "Conjugate Input Type." The modulation product frequencies are distributed as shown in Fig. 9*a*. When both signal and carrier are impressed on the series branch the modulation product frequencies are

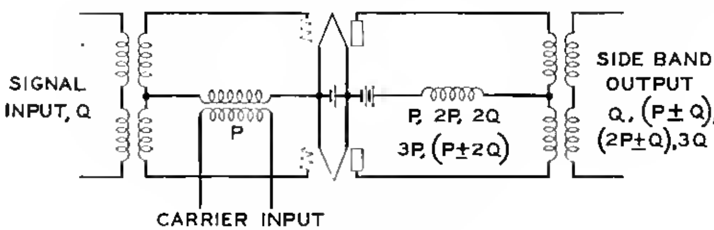


Fig. 9*a*. Conjugate Input Type

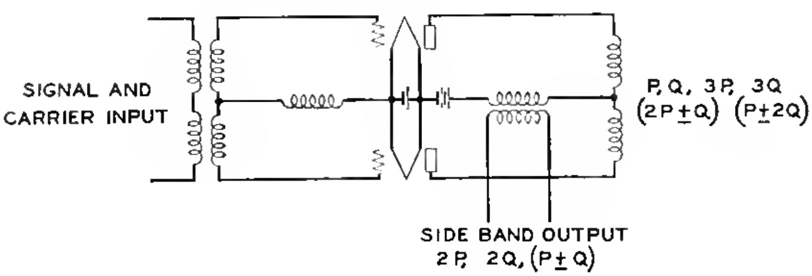


Fig. 9*b*. Common Input Type

as shown in Fig. 9*b* and the circuit is called for convenience the "Common Input Type." The phase of the modulation product of any order may be determined from the consideration that the phase of the frequency

$$\frac{1}{2\pi}|mp \pm nq|$$

depends upon the quantity $(m\theta_p \pm n\theta_q)$ where θ represents the phase

angle between current and voltage of each input frequency. This conclusion is independent of the type of modulation employed. If the phase of the product is then calculated to be identical on the two grids or two plates, it appears in the midbranch; if it turns out to be opposite in phase on the two grids referred to the filament, it appears in the series arm.

Conjugate Input Grid Modulator

With the signal introduced in the series arm of Fig. 9*a*, the sideband potential is built up across the same arm, so that a high sideband impedance must be provided by the input transformer terminated in its filter,—a low pass filter for the modulator, and a band pass filter for the demodulator. The carrier frequency is introduced in the conjugate arm so that the carrier circuit does not directly affect the signal and sideband impedances. There is a second order effect, however, due to the reaction of those modulation products which flow in the common branch. The input impedance may be expected to change also when the coupling between the two high impedance windings of the input transformer is varied, since this effectively changes the impedance to the above mentioned modulation products.

Modulation is largely eliminated in the plate circuit and the load capacity of the tubes is increased by inserting a choke coil in the common plate branch to suppress the carrier current. This, incidentally, tends to reduce carrier leak. The impedance of the choke coil at the carrier frequency is modified by the capacity to ground of the output transformer, and must be designed with this point in mind since the shunt arm impedance may otherwise be materially reduced. Some further increase in load capacity is obtained by having the output transformer and the terminating filter offer a high impedance to frequencies outside the transmitted band. In this way all important components except the sideband are suppressed and the plate circuit of the tube operates as an amplifier so that the plate power dissipation is reduced and the load capacity increased. The same considerations regarding the second order impedance effects of the shunt branch on the series branch exist for the plate circuit as for the grid circuit considered above. The main effect when there is loose coupling between the high impedance windings of the output transformer is to introduce an inductive reactance into the series arm. This tends to increase the reflection coefficient so that it becomes preferable to couple the two windings closely, a comparatively easy thing to do in low impedance circuits. The modulation products accompanying the desired product are indicated in Fig. 9, and it is seen that there will be no introduced distortion up to the third order when the carrier frequency is sufficiently high.

Common Input Grid Modulator

Another useful type of grid current modulator is shown in Fig. 9*b*, in which both signal and carrier are applied across the same input terminals. The modulation currents flow in the plate (and corresponding grid) circuits as shown in the above schematic. Where the ratio of carrier to signal frequency is large so that a single input transformer cannot be used efficiently, separate transformers with associated filter networks may be used for each of the two inputs. Since the second order sidebands ($p \pm q$) appear in the midbranches, it is not necessary to have the impedance high to these frequencies in the input coil, but only from the midpoint of the input coil to ground. This is most conveniently accomplished by a high inductance retard coil in the midbranch of the grid circuit, although transformers and high impedance networks may be used in general. The grid circuit sideband across the midbranch is amplified and appears in the plate circuit midbranch. The fundamental currents together with all odd order modulation products are eliminated by a high impedance, high mutual retard coil in the series arm of the plate circuit.

Since the present practice is to use suppressed carrier, a hybrid [11] coil must be used to introduce the carrier if this circuit is to be used as a demodulator, although the signal and carrier currents may be introduced through filters when used as a modulator. Either frequency discrimination or balance is required in any case to keep carrier current out of the signal circuit.

The chief advantage of the common over the conjugate input type of circuit is that the high impedance required for the modulated product is provided by a distinct element, and no high impedance requirements are placed on other elements in either input or output circuits. Another advantage of this arrangement is that the amplified fundamentals are balanced out, making the singing gain about 20 T.U. less than that of the conjugate input type. The only modulation products (up to the fourth order) not balanced out of the output are the second harmonics of carrier and signal. This type of circuit may be used as a demodulator at any frequency, but as a modulator only when the second harmonic of the highest voice frequency does not come in the sideband range—it is therefore not well adapted to modulate low carrier frequencies where high quality is required.

Although the output of this modulator is affected but little by the filter impedance in either input or output circuits, some care is neces-

[11] By using a hybrid coil having eight times as many turns in the signal circuit as in the carrier circuit, the equivalent current losses to signal and carrier are 0.5 T.U. and 9.5 T.U. respectively instead of 3 T.U. each, as is the case for the usual equality ratio hybrid coil.

sary in selecting the retard coils for the grid and plate circuits. Since the grid retard should have a high impedance to the desired modulation frequencies it must have an inductance of the order of 50 henries or greater at low frequencies in a demodulator. Resonance in the voice band is not harmful so long as the impedance does not drop too much at high voice frequencies.

The plate circuit retard coil is well balanced to reduce the unbalanced carrier transmitted to the line. An important requirement is that of close coupling so that the reactance in the output circuit may not be great enough to cause a transmission loss or large reflection coefficient. The required inductance then depends upon the relative separation of voice and sideband frequencies. If the lowest sideband frequency is very close to the highest voice frequency it may be impossible to prevent positive reactance from coming into the voice circuit of a demodulator, but the effect may be considerably reduced by utilizing this positive reactance in the mid-series section of the adjacent low pass filter.

Double Balanced Circuits

If two balanced circuits of either of the above types are connected with their input and output terminals respectively in series, all the modulation products up to the fourth order except the second order sidebands may be balanced out. There is no hybrid or filter loss and due to more complete suppression of unwanted frequencies the maximum output power obtainable is more than twice that with a single balanced circuit. The complexity of the resultant circuit is such as to rule it out for all ordinary applications.

For purposes of comparison we proceed to consider the experimental results obtained on a conjugate input grid modulator designed in accordance with the ideas set forth above.

Experimental Results

Figs. 10 and 11 represent the results of experiment on a conjugate input grid modulator with a carrier frequency of 6,800 cycles and a signal frequency of 1,000 cycles. The input and output networks previously discussed and represented in Figs. 6 and 7 were used here with 101-D tubes operated at 120 volts plate potential and 1.0 ampere filament current. The grids were connected to the negative terminal of the filaments. Fig. 10 represents the sideband output current in a 675 ohm circuit, plotted as a function of the signal current measured in the 675 ohm input circuit, with the carrier input maintained at 15 mils throughout. The upper four curves represent various experimental conditions designed to bring out the effect of different circuit

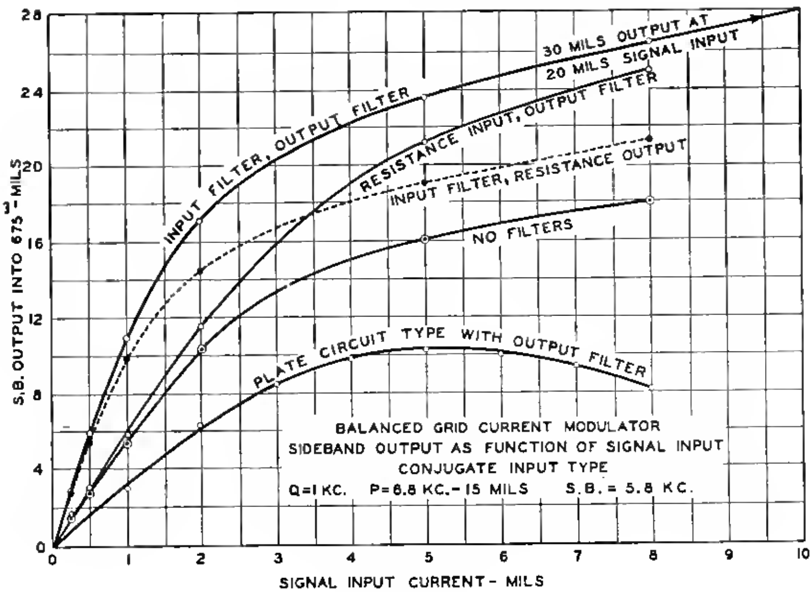

30 MILS OUTPUT AT
20 MILS SIGNAL INPUT
INPUT FILTER, OUTPUT FILTER
RESISTANCE INPUT, OUTPUT FILTER
INPUT FILTER, RESISTANCE OUTPUT
NO FILTERS
PLATE CIRCUIT TYPE WITH OUTPUT FILTER
BALANCED GRID CURRENT MODULATOR
SIDEBAND OUTPUT AS FUNCTION OF SIGNAL INPUT
CONJUGATE INPUT TYPE
Q=1 KC. P=6.8 KC.-15 MILS S.B.=5.8 KC.
S.B. OUTPUT INTO 675 - MILS
SIGNAL INPUT CURRENT - MILS

Fig. 10

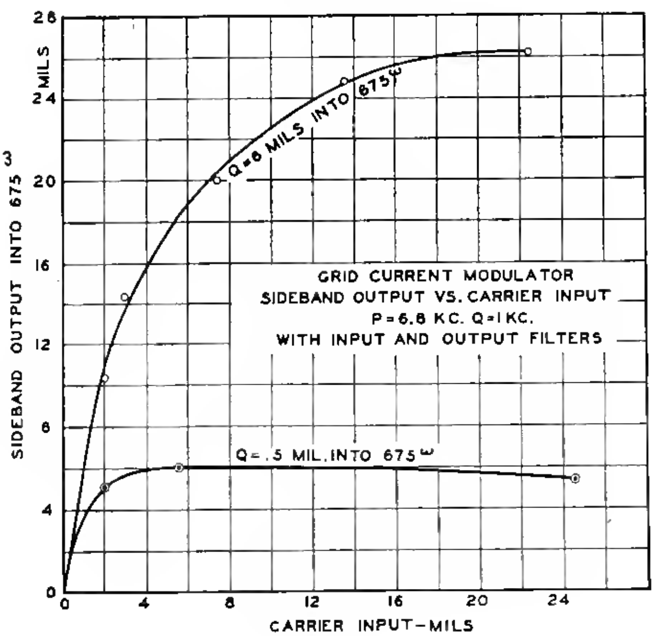

Q=6 MILS INTO 675
GRID CURRENT MODULATOR
SIDEBAND OUTPUT VS. CARRIER INPUT
P=6.8 KC. Q=1 KC.
WITH INPUT AND OUTPUT FILTERS
Q=.5 MIL. INTO 675
SIDEBAND OUTPUT INTO 675 MILS
CARRIER INPUT-MILS

Fig. 11

elements, while the lowest curve illustrates the performance of a representative conjugate input plate modulator working under conditions prescribed for it into a 600 ohm circuit with the same tubes and plate potential. A direct comparison between the two types of modulator as to sideband current output should include a comparative increase of 0.6 T.U. to the grid current modulator output to take care of the difference in the two load impedances.

The curve labelled "no filters" applies to the circuit of Fig. 9 in which both input and output circuits were connected to 675 ohm resistances. The presence of the retard coil in the plate circuit is accountable for the increase in output at large signal inputs over that of the plate type. When an output filter is added (resistance input, output filter) the gain at low inputs is scarcely affected but the output power for large signals is doubled since the load capacity is increased by the suppression of the signal frequency current in the plate circuit. If now an input filter is inserted and the output connected to a 675 ohm circuit (input filter, resistance output) the gain at low signal currents is increased by about 5 T.U. over that with no filters in circuit, while the increase at high signal amplitudes is of the order of 1.5 T.U. The topmost curve represents the performance of the modulator circuit terminated in the two filters, which shows a modulating gain of 21.5 T.U. at small inputs and a maximum power output of 30 mils into 675 ohms (0.6 watt). Fig. 11 represents the effect of varying the carrier input at two signal inputs—0.5 and 6 mils respectively. This illustrates the lack of dependence of sideband on carrier when the carrier is greater than the signal, which was deduced from eq. 20 as characteristic of this type of modulator. The use of a 15 mil carrier is seen to furnish close to the optimum value for the circuit, at least when the signal amplitude does not greatly exceed 6 mils.

The common input type is capable of yielding much the same results as the conjugate input type with somewhat less care required for the flanking filter impedances, since the proper circuit impedances are obtained by the use of retard coils as shown in Fig. 9*b*. On theoretical grounds, however, as we mentioned in discussing the general properties of balanced circuits, it is not capable of furnishing as high quality as the conjugate type at the low sideband frequency used here. At high sideband frequencies this objection disappears, so that the reduced filter requirements make it perhaps more attractive in application than the conjugate type. It should be noted that the plate modulator may be made to have a greater gain than that shown in Fig. 10 by changing the input transformer (with the same maximum output level) but this restricts the signal input current to correspondingly smaller amplitudes.

A few words on the shape of the signal-sideband curve of plate modulators of the van der Bijl type may not be inappropriate at this point since the curve depends to some extent upon the incidental grid modulation produced. Thus at large signal amplitudes the grid of the modulator tube is driven positive and grid modulation is produced, which tends to oppose plate modulation. By reversing the conditions which we have employed in the grid current modulator to promote grid modulation, the net plate modulation may be increased and the sideband-signal curve may more nearly show an asymptotic maximum which is so desirable from the overloading standpoint. This condition is evidently secured with a flanking input filter having a low impedance to the sideband, or by having an input coil which, while not seriously affecting the transmission of signal frequencies, offers of itself a low impedance to the grid sideband. Thus in plate modulators the input coil would have a high winding capacity, and in plate demodulators it would have comparatively low mutual inductance between primary and secondary windings.

As an indication of the quality obtained with the grid current modulating process, comparative listening tests between carrier telephone systems employing plate and grid demodulators, respectively, conducted by R. W. Chesnut, indicate roughly a 10 T.U. greater load carrying capacity for the grid type over a wide range of input amplitudes at about the same quality in both cases. The carrier leak may be reduced to one half mil by a not very critical tube selection, which is quite satisfactory in general.

The last point remaining is the plate power efficiency, which we have defined as the ratio of the sideband power developed in the load resistance to the d.c. power supplied to the plate circuit under operating conditions—it is really the efficiency of power conversion. At maximum output it is three per cent for the standard plate modulator and fifteen per cent for the above grid modulator. The efficiencies obtained at maximum output for a number of different low power tubes used in the grid current modulator may be tabulated as follows:

Tube	Plate Potential	Sideband Power W_{AC}	Plate Efficiency W_{AC}/W_{DC}
230-D.......	60	0.022	11%
221-A.......	70	0.065	18
221-D.......	90	0.13	14
101-D.......	120	0.50	15
102-D.......	120	0.11	22

For design information and construction of the experimental models,

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Appendix

Grid Current Modulator, Large Grid Potentials

Making use of the observation that the positive lobes of the input wave are effectively suppressed with a sufficiently large external grid resistance, we first define a function equal to zero when the independent variable is positive, and equal to the variable when the variable is negative. This is evidently a representation of the potential effective on the grid in terms of the applied potential. If we denote the grid potential by $-f(y)$ where y is the impressed potential, it may be expressed as a Fourier series

$$-f(y) = b_0/2 + \Sigma b_m \cos m\pi y/Y + a_m \sin m\pi y/Y, \tag{14}$$

in which the coefficients are determined by the usual relations

$$\left.\begin{aligned} b_m &= \frac{1}{Y}\int_0^Y y \cos\frac{m\pi y}{Y}dy = \frac{Y}{m^2\pi^2}(\cos m\pi - 1), \\ a_m &= \frac{1}{Y}\int_0^Y y \sin\frac{m\pi y}{Y}dy = (-1)^{m-1}\frac{Y}{m\pi}\cos m\pi, \\ b_0 &= \frac{1}{Y}\int_0^Y ydy = Y/2. \end{aligned}\right\} \tag{15}$$

In these equations y represents the generator e.m.f. and Y is its maximum value. If we put (15) in (14), we have

$$\begin{aligned} -f(y) = Y/4 - \frac{2Y}{\pi^2}\sum_{m=1}^{m=\infty}\frac{\cos(2m-1)\pi y/Y}{(2m-1)^2} \\ + \frac{Y}{\pi}\sum_{m=1}^{m=\infty}\frac{(-1)^{m+1}}{m}\sin\frac{m\pi y}{Y}, \end{aligned} \tag{16}$$

the last term of which may be summed to $y/2$ and (16) may be rewritten as

$$-f(y) = Y/4 + y/2 - \frac{2Y}{\pi^2}\sum_{m=1}^{m=\infty}\frac{\cos(2m-1)\pi y/Y}{(2m-1)^2}, \tag{17}$$

which represents the desired solution. It will be observed that the first two terms of the right member contribute only a d.c. term and the fundamentals, so that the other modulation products must come from the summation. This expression is a perfectly general one as far as the form of y is concerned. In the case of a sinusoidal grid e.m.f. it is possible by more customary methods to find the grid potential, but

with a complex grid potential in which we are primarily interested no simpler representation is known to the authors except where the two frequencies involved are harmonically related.

With the two frequency inputs considered, we have the grid potential

$$y = P \cos pt + Q \cos qt \tag{18}$$

and the summation term may be written

$$\frac{2(P+Q)}{\pi^2}\sum_{m=1}^{m=\infty}\frac{\cos\left[(2m-1)\pi(P\cos pt + Q\cos qt)/(P+Q)\right]}{(2m-1)^2}. \tag{19}$$

Upon expansion of (19) as the cosine of the sum of two angles we find the terms $\cos(A\cos\theta)$ and $\sin(A\cos\theta)$ which require evaluation before the solution can be put in significant form. This might conceivably be done by direct expansion; for example, the first of the expressions would then be

$$\cos(A\cos\theta) = 1 - \frac{(A\cos\theta)^2}{2!} + \frac{(A\cos\theta)^4}{4!}\cdots.$$

Putting the terms of this series in terms of multiple angles gives us an infinite series to be summed as the coefficient of each multiple angle term. This may be done in terms of Bessel coefficients by Jacobi's expansions [12] which are as follows

$$\cos(A\cos\theta) = \sum_{n=0}^{n=\infty}(-1)^n\epsilon_{2n}J_{2n}(A)\cos 2n\theta,$$

$$\sin(A\cos\theta) = \sum_{n=0}^{n=\infty}(-1)^n\epsilon_{2n+1}J_{2n+1}(A)\cos(2n+1)\theta,$$

in which $J_k(A)$ is a Bessel coefficient of the kth order and ϵ_k is Neumann's factor which is two for k not zero, and unity for k zero. Carrying out the expansion, the sideband amplitude comes out as

$$\frac{4(P+Q)}{\pi^2}\left[J_1\left(\frac{P\pi}{P+Q}\right)J_1\left(\frac{Q\pi}{P+Q}\right) + \frac{1}{3^2}J_1\left(\frac{3P\pi}{P+Q}\right)J_1\left(\frac{3Q\pi}{P+Q}\right) + \cdots\right], \tag{20}$$

which may be computed from tables to be found in Watson's book [13] previously cited. Analogous expressions exist for the other components.

[12] Watson, "Theory of Bessel Functions," p. 22.
[13] P. 666 et seq.